Preliminary Mathematics Subject Matter Requirements

Domain 1. Algebra

Candidates demonstrate an understanding of the foundations of the algebra contained in the Mathematics Content Standards for California Public Schools as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of algebra and its underlying structures, candidates must have a deep conceptual knowledge. They are skilled at symbolic reasoning and use algebraic skills and concepts to model a variety of problem-solving situations. They understand the power of mathematical abstraction and symbolism.

1.1 Algebraic Structures

a. Show that the real and complex numbers are each a field, and explain why particular rings are not fields (e.g., polynomial and matrix rings, modular systems)
b. Apply field properties in constructing mathematical arguments (e.g., if $a < b$ and $c < 0$, then $ac > bc$)
c. Know that the rational numbers and real numbers are ordered fields and that the complex numbers are not an ordered field, but that they are the algebraic closure of the real numbers

1.2 Polynomial Equations and Inequalities

a. Know why graphs of linear inequalities are half planes and apply this fact to linear programming
b. Prove and use the following:
   - The Rational Root Theorem for polynomials with integer coefficients
   - The Factor Theorem
   - The Conjugate Roots Theorem for polynomial equations with real coefficients
   - The Quadratic Formula for real and complex quadratic polynomials
   - The Binomial Theorem
c. Analyze and solve polynomial equations with real coefficients using:
   - Descartes' rule of signs
   - The Fundamental Theorem of Algebra
   - The Roots of Unity (the roots of $x^n = 1$ are points on the unit circle and they form the vertices of a regular n-sided polygon)
d. Know that cubic and quartic polynomial equations are solvable by radicals, but that polynomial equations of degree 5 or higher are not

1.3 Functions

a. Analyze and prove general properties of functions (i.e., domain and range, one-to-one, onto, inverses, composition, and differences between relations and functions)
b. Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems)

c. Analyze properties of exponential and logarithmic functions in a variety of ways (e.g., graphing, solving problems)

1.4 Linear Algebra

a. Understand and apply the geometric interpretation and basic operations of vectors in two or three dimensions, including their scalar multiples and scalar (dot) and cross products
b. Prove the basic properties of vectors described in 1.4a
c. Understand, apply, and prove the basic properties and operations of matrices and determinants (e.g., the solvability of linear systems of equations)

Domain 2. Geometry

Candidates demonstrate an understanding of the foundations of the geometry contained in the Mathematics Content Standards for California Public Schools as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of geometry and its underlying structures, candidates must have a deep conceptual knowledge. They demonstrate an understanding of axiomatic systems and different forms of logical arguments. Candidates understand, apply, and prove theorems relating to a variety of topics in two- and three-dimensional geometry, including coordinate, synthetic, non-Euclidean, and transformational geometry.

2.1 Parallelism

a. Know the Parallel Postulate and its implications, and justify its equivalents (e.g., the Alternate Interior Angle Theorem, the angle sum of every triangle is 180 degrees)
b. Know that simple variants of the Parallel Postulate produce two different non-Euclidean geometries (i.e., spherical and hyperbolic)

2.2 Plane Euclidean Geometry

a. Prove theorems and solve problems involving similarity and congruence
b. Understand, apply, and justify properties of triangles (e.g., the Exterior Angle Theorem, concurrence theorems, trigonometric ratios, Triangle Inequality, Law of Sines, Law of Cosines, the Pythagorean Theorem and its converse)
c. Understand, apply, and justify properties of polygons and circles from an advanced standpoint (e.g., derive the area formulas for regular polygons and circles from the area of a triangle)
d. Justify and perform the classical constructions (e.g., angle bisector, perpendicular bisector, replicating shapes, regular n-gons for n equal to 3, 4, 5, 6, and 8)
e. Use techniques in coordinate geometry to prove geometric theorems (e.g., deduce properties of conic sections from their equations)

2.3 Three-Dimensional Geometry

a. Prove and use parallelism and perpendicularity of lines and planes in three dimensions

b. Understand, apply, and justify properties of three-dimensional objects from an advanced standpoint (e.g., derive the volume and surface area formulas for prisms, pyramids, cones, cylinders, and spheres)
c. Understand and verify Euler's formula for polyhedra

2.4 Transformational Geometry

a. Understand and prove basic properties of isometries in two- and three-dimensional space (e.g., isometries form a group under composition, every isometry is a composition of reflections)
b. Understand and prove the basic properties of dilations

Domain 3. Number Theory

Candidates demonstrate an understanding of the number theory and a command of the number sense contained in the Mathematics Content Standards for California Public Schools as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of number theory and its underlying structures, candidates must have a deep conceptual knowledge. They prove and use properties of natural numbers. They prove results and solve problems in modular systems. They formulate conjectures about the natural numbers using inductive reasoning, and verify conjectures with proofs.

3.1 Natural Numbers

a. Prove and use basic properties of natural numbers (e.g., properties of divisibility)
b. Use the Principle of Mathematical Induction to prove results in number theory
c. Prove and use the Euclidean Algorithm
d. Apply the Fundamental Theorem of Arithmetic (e.g., find the greatest common factor and the least common multiple, show that every fraction is equivalent to a unique fraction where the numerator and denominator are relatively prime, prove that the square root of any number, not a perfect square number, is irrational)

3.2 Modular Systems

a. Prove basic results about congruence modulo n
b. Compute and solve problems in modular systems (e.g., ISBN, UPC, check digits, the Chinese Remainder Theorem)
c. Solve and apply linear Diophantine equations

Domain 4. Probability and Statistics

Candidates demonstrate an understanding of the statistics and probability distributions for advanced placement statistics contained in the Mathematics Content Standards for California Public Schools as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of probability and statistics and its underlying structures, candidates must have a deep conceptual knowledge. They solve problems and make inferences using statistics and probability distributions.

4.1 Probability

a. Prove and apply basic principles of permutations and combinations
b. Illustrate finite probability using a variety of examples and models (e.g., the fundamental counting principles)
c. Use and explain the concept of conditional probability
d. Interpret the probability of an outcome (e.g., the area of the region under the graph of the probability density function of a continuous random variable)
e. Use normal, binomial, and exponential distributions to solve and interpret probability problems

4.2 Statistics

a. Compute and interpret the mean, median, and mode of both discrete and continuous distributions
b. Compute and interpret quartiles, range, variance, and standard deviation of both discrete and continuous distributions
c. Select and evaluate sampling methods appropriate to a task (e.g., random, systematic, cluster, convenience sampling) and display the results
d. Prove the method of least squares and apply it to linear regression and correlation
e. Know and apply the chi-square test

Domain 5. Calculus

Candidates demonstrate an understanding of the trigonometry and calculus contained in the Mathematics Content Standards for California Public Schools as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of trigonometry and calculus and their underlying structures, candidates must have a deep conceptual knowledge. They apply the concepts of trigonometry and calculus to solving problems in real-world situations.

5.1 Trigonometry

a. Prove that the Pythagorean Theorem is equivalent to the trigonometric identity $\sin^2 x + \cos^2 x = 1$ and that this identity leads to $1 + \tan^2 x = \sec^2 x$ and $1 + \cot^2 x = \csc^2 x$
b. Prove the sine, cosine, and tangent sum formulas for all real values, and derive special applications of the sum formulas (e.g., double angle, half angle)
c. Analyze properties of trigonometric functions in a variety of ways (e.g., graphing and solving problems)
d. Analyze properties of inverse trigonometric functions (i.e., arcsin, arcos, and arctan) in a variety of ways (e.g., graphing and solving problems).
e. Understand and apply polar representations of complex numbers (e.g., DeMoivre's Theorem)

5.2 Limits and Continuity

a. Derive basic properties of limits and continuity, including the Sum, Difference, Product, Constant Multiple, and Quotient Rules, using the formal definition of a limit
b. Show that a given function is continuous at a point, and distinguish between removable and non-removable discontinuities
c. Know and apply the Intermediate Value Theorem, using the geometric implications of continuity

5.3 Derivatives and Applications

a. Derive the rules of differentiation for algebraic, trigonometric, inverse trigonometric, logarithmic, and exponential functions using the formal definition of derivative
b. Interpret the concept of derivative geometrically, numerically, and analytically (i.e., slope of the tangent, limit of difference quotients, extrema, Newton's method, and instantaneous rate of change)
c. Interpret continuous, differentiable functions geometrically and analytically and apply Rolle's Theorem, the Mean Value Theorem, and L'Hopital's rule
d. Use the derivative to solve rectilinear motion, related rate, and optimization problems
e. Use the derivative to analyze functions and planar curves in rectangular and parametric form
f. Solve separable first-order differential equations and apply them to growth and decay problems
g. Know and apply the concept of a partial derivative

5.4 Integrals and Applications

a. Derive definite integrals of standard algebraic functions using the formal definition of integral
b. Interpret the concept of a definite integral geometrically, numerically, and analytically (e.g., limit of Riemann sums, improper integrals)
c. Prove the Fundamental Theorem of Calculus, and use it to interpret integrals as antiderivatives
d. Apply the concept of integrals to compute the length of curves and the areas and volumes of geometric figures
e. Know and apply the concept of multiple integrals

5.5 Sequences and Series

a. Derive and apply the formulas for the sums of finite arithmetic series and finite and infinite geometric series (e.g., express repeating decimals as a rational number)
b. Use Ratio, Comparison, and Integral Tests to test convergence on infinite series (e.g., show that a series may be divergent even if the n-th term goes to zero, find the interval of convergence by Ratio Test)
c. Derive and apply Taylor series and Taylor polynomials of basic functions (e.g., binomial series)

Domain 6. History of Mathematics

Candidates understand the chronological and topical development of mathematics and the contributions of historical figures of various times and cultures. Candidates know important mathematical discoveries and their impact on human society and thought. These discoveries form a historical context for the content contained in the Mathematics Content Standards for California Public Schools as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) (e.g., numeration systems, algebra, geometry, calculus).

6.1 Chronological and Topical Development of Mathematics

a. Demonstrate understanding of the development of mathematics, its cultural connections, and its contributions to society
b. Demonstrate understanding of the historical development of mathematics, including the contributions of diverse populations as determined by race, ethnicity, culture, geography, and gender